The cosmic censor conjecture: Is it generically violated?

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It has been recently argued by Hertog, Horowitz and Maeda [1], that generic reasonable initial data in asymptotically anti deSitter, spherically symmetric, space-times within an Einstein-Higgs theory, will evolve toward a naked singularity, in clear violation of the *cosmic censor conjecture*. We will argue that there is a logical and physically plausible loophole in the argument and that the numerical evidence in a related problem suggests that this loophole is in fact employed by physics.

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I. INTRODUCTION

One of the most famous conjectures of gravitational physics and perhaps of all physics is the so called *cosmic censor conjecture* (CCC) [2]. The physical formulation of this conjecture states that, for physically reasonable initial data, space-time cannot evolve toward a naked singularity. That is, if a singularity forms, it will be covered by an event horizon (*i.e.* it will be contained within a black hole), indicating that far away observers will not be influenced by it. The importance of the conjecture resides in the fact that its validity will prevent the complete collapse of the predictive power of physical laws (a naked singularity will eventually influence the rest of the space-time), as far as asymptotic observers are concerned (among which we count ourselves). On the other hand, singularity theorems predict the formation of singularities starting from certain regular initial data when appropriate energy conditions are satisfied. However, up to now, an despite the efforts of many relativists, it has not been possible to prove that such singularities will always be contained within a black hole (the conjecture resists to be promoted to the status of a theorem).

Most of the reasonable matter that undergoes gravitational collapse ends up in the formation of a black hole with a completely regular domain of outer communication, a fact that produces strong supporting evidence for the validity of the CCC. On the other hand, there are indeed "counterexamples" that clearly violate the CCC. However, these are not generic in the sense that the initial conditions leading to the formation of a naked singularity are fine-tuned [3]. Since it is thought to be physically impossible to prepare a system with such precise initial conditions, those counterexamples are considered to be rather artificial, and the CCC is nowadays considered to apply only in the generic sense.

Recently, however, Hertog, Horowitz and Maeda (HHM) [1], reported having found a generic counterexample to the CCC. To that end, they construct an open set of initial data within an Einstein-Higgs system in asymptotically anti deSitter (AdS), spherically symmetric, space-times with a scalar-field potential $V(\phi)$ which is not positive semi-definite. In this report, we re-analyze the situation and show that the HHM arguments have a generic loophole, which we describe in a general setting. Furthermore, we will argue that although at this point our analysis does not prove or disprove the example constructed in [1], the existing numerical evidence points toward the realization of our generic loophole rather than the formation of a naked singularity.

II. DESCRIPTION

The situation considered in [1] corresponds to a scalar field minimally coupled to gravity with Lagrangian

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{16\pi}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] . \qquad (2.1)$$

(units where $G = c = 1$ are employed).

The scalar field has a tilted Mexican hat potential with a true minimum at $\phi = \phi_a$ with $V(\phi_a) = -a$, and a local minimum at $\phi = \phi_b$ with $V(\phi_b) = -b$ ($a > b > 0$). The idea is to construct initial data that will evolve toward a cosmological type singularity in a central region, while having a mass that is to small to allow the formation of a black hole sufficiently large to enclose the singularity. To construct this initial data one needs to provide on a 3-manifold the 3-metric h_{ab}, the extrinsic curvature K_{ab}, the scalar field ϕ and its time derivative, all subject to the momentum and hamiltonian constraints. The strategy to construct the example that is argued to lead to a naked singularity is as follows:

One considers the manifold $\mathbb{R}^3$ and constructs spherically symmetric initial data with $K_{ab} = 0$ and $\partial_t\phi = 0$. The tree metric can be written as

$$h_{ab}dx^a dx^b = A(r)dr^2 + B(r)r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) . \qquad (2.2)$$

To find the metric function $A(r)$ one needs only be concerned with the hamiltonian constraint which, upon the

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use of the following re-parametrization

$$A \equiv \left(1 - \frac{2m(r)}{r} - \frac{\Lambda_{\rm eff} r^2}{3}\right)^{-1}, \tag{2.3}$$

can be cast simply as

$$m' = 4\pi r^2 \left[\frac{1}{2A}{\phi'}^2 + V(\phi) + b\right]. \tag{2.4}$$

In the case of interest we want to consider a field configuration that interpolates from a central region with $\phi = \phi_a$ to an exterior region with $\phi = \phi_b$. We then choose an effective cosmological constant $\Lambda_{\rm eff} = -8\pi b$, thus ensuring the convergence of $M = \lim_{r\to\infty} m(r)$ (provided that asymptotically $\phi \sim \phi_b + Cr^{-3/2-|\epsilon|}$). Given the scalar field configuration we can solve for $m(r)$ from the hamiltonian constraint above [1] . The initial data is then completely specified by $\phi(r)$.

One now looks for a class of configurations that would have $\phi(r) = \phi_a + \epsilon$ for $r < R_0$, and $\phi(r) = \phi_b$ for $r > R_1$ ($R_0 < R_1$), with R_0 scaling more or less linearly with R_1 within the class, and with a total mass that also scales linearly with R_1. The argument would then go as follows: In the domain of dependence corresponding to the region with $r < R_0$ we will have a cosmological solution corresponding to anti-De-Sitter space-time with a scalar field oscillating at the bottom of the potential, which is known to lead to a singularity within a finite proper time as seen by co-moving observers. Moreover, we can ensure that there are points on the initial surface for which all future directed causal curves emanating from them will hit the singularity.

Now let us assume that the evolution of the initial data produces a black hole that encloses the singularity. The black hole will eventually settle to a stationary stage which must correspond to a Schwarzschild anti-De-Sitter (SAdS) space-time with cosmological constant corresponding to the asymptotic value of the potential $\Lambda_{\rm eff} = -8\pi b$, and whose mass can not exceed the mass available in the initial configuration. This limits the area of this late time black hole and, given the fact that the area of the event horizon is a non-decreasing quantity, it bounds the area of the region that can be enclosed by any horizon that might be present at the initial hypersurface. The mass has been arranged to scale with R_1 which implies that the bound on the area scales as $R_1^{1/3}$ (at least for large enough SAdS holes). For the singularity to be enclosed by the event horizon, that null surface must intersect either 1) the boundary of the domain of dependence of the initial data region with $r < R_0$ (*i.e.* the in-going null congruence starting at $r = R_0$ before the generation of the singularity), or 2) the initial hypersurface at $r > R_0$. Both these cases would require the area of the horizon to exceed its bound. This will preclude the singularity from been enclosed by such horizon, and thus the singularity must be naked.

The loophole lies in the assumption that the space-time will evolve toward a stationary configuration at late times. In fact, it is quite possible in principle for the following alternative scenario to develop: The outermost region corresponding to the asymptotic regime of the SAdS solution with $\Lambda_{\rm eff} = -8\pi b$ is continuously shrinking (from the inside) due to a domain wall which expands outward and which corresponds to the scalar field interpolating between the two minima. The inner region becomes a SAdS spacetime with a large black hole that encloses the singularity and which would, if extending infinitely outward, have a mass that exceeds the initial mass of the space-time, but we must recognize that here we are dealing with two very different notions of mass corresponding to two different values of the cosmological constant for the initial and final AdS space-times one would be considering ($\Lambda^a = -8\pi a$ and $\Lambda^b = -8\pi b = \Lambda_{\rm eff}$). Of course, in reality the interior AdS region never covers the whole space-time. The wall would behave as a solitonic type solution (such as a boosted "kink" in Sine-Gordon system) in that, once initially set it would always keep moving outward (in our case it could, of course, be changing its form and needs not be a true soliton). The apparent paradoxical situation which would allow the generation of a large black hole which will be present in any late time hypersurface is explained as due to the large negative energy which, relative to AdS with cosmological constant Λ^b, represents the large inner region with cosmological constant Λ_a.

III. NUMERICAL EXAMPLE

We have performed a numerical simulation of a related system in which the local minimum of the potential corresponds to zero vacuum energy ($b = 0$), which replaces the asymptotic AdS region by an asymptotically flat region and makes the numerical evolution considerably less cumbersome. Our potential has the form (see Fig. 1)

$$V = \frac{1}{4}\phi^2\left(\phi^2 - \frac{4}{3}(\eta_1 + \eta_2)\phi + 2\eta_1\eta_2\right), \tag{3.1}$$

with $\eta_1 = 0.5$ and $\eta_2 = 0.1$.

For this potential, we have constructed a static unstable soliton as in Ref. [4]. We later perturb this initial configuration in the form $\partial_t\phi(0,r) = \epsilon e^{-r^2}$ ($\epsilon << 1$). We also specify $B(0,r) = 1$, and solve the hamiltonian constraint for $A(0,r)$ and the momentum constraint for $K_{ab}(0,r)$. As mentioned, since the static soliton configuration is unstable, the small perturbation in the scalar field suffices to trigger dynamical evolution away from the initial configuration. Here the sign for ϵ is chosen for the configuration to "explode" rather than to collapse to a black hole [5], since we want to simulate a situation in which the spacetime does not settle to a stationary state. The dynamics of the spacetime is followed by solving the full non-linear 3+1 Einstein evolution equations [6, 7] for the spherically symmetric case (see Ref. [5] for details). For the simulation considered here we have used Eulerian

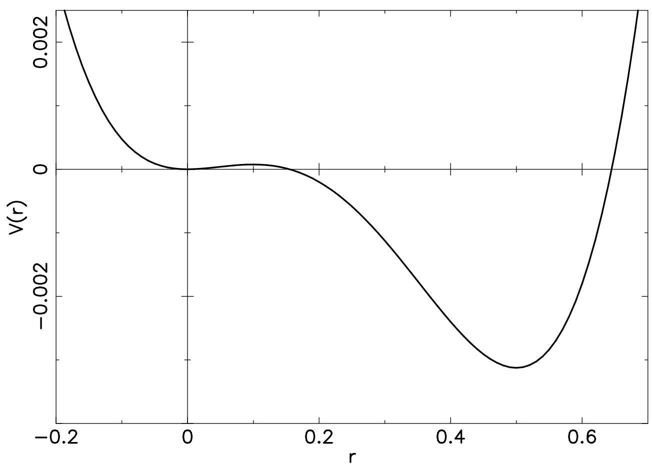

FIG. 1: Scalar potential $V(\phi)$ corresponding to Eq. (3.1).

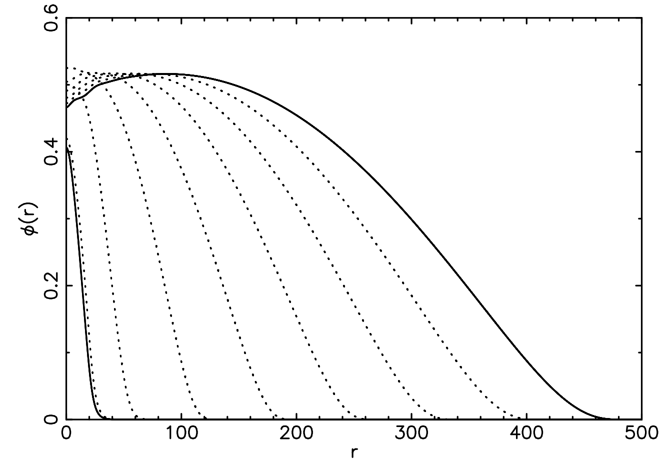

FIG. 2: Evolution of the scalar field ϕ. Notice how the field is moving toward the true minimum at $\phi = 0.5$ everywhere.

spatial coordinates (*i.e.* zero shift vector) and harmonic time slicing, which should approach the interior singularity asymptotically reaching it only after an infinite coordinate time [8, 9] (the lapse function collapses to zero as the singularity is approached).

The numerical simulation shows that the scalar field tends to move to the global minimum of the potential everywhere, with a scalar field "wall" moving outward. The result of this is that the outer Schwarzschild regions (where $V(\phi) \sim 0$) are eventually reached by the wall of scalar field, becoming inner AdS regions (where $V(\phi) < 0$). Therefore, the final configuration never reaches a stationary state. Figures 2-4 depict this evolution sequentially from the initial configuration to the point where the moving wall reaches the outer boundary of the numerical grid (initial and "final" states are shown by solid lines, intermediate stages by dashed lines). The numerical evolution eventually freezes when the lapse function collapses everywhere to zero. We have looked for apparent horizons at every time step during the evolution but have found none. Notice that the fact that no apparent horizon seems to form does not imply that no black hole is present, as an event horizon might very well exist. The presence of an event horizon seems likely, as outgoing null lines outside the scalar field wall should reach null infinity, while those inside should reach the singularity instead. However, should such an event horizon exist, the black hole will not reach a stationary state and would eventually swallow the whole spacetime.

Our simulation shows what will probably happen in the true SAdS situation. Thus, we conjecture that the corresponding spacetime will not reach a stationary situation. We emphasize that the numerical simulation for SAdS is considerably more difficult than in the asymptotically flat case for reasons related to numerical stability in the exterior regions. However, we are currently working around the stability problem and will report on this in the near future.

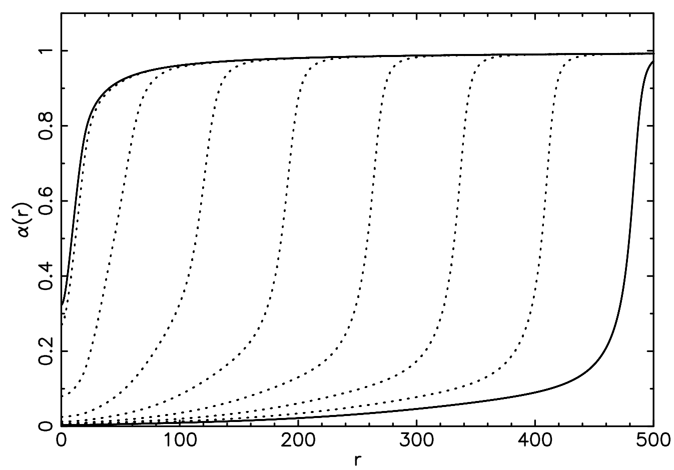

FIG. 3: Evolution of the lapse function α. Notice the collapse of the lapse indicating the approach to a singularity.

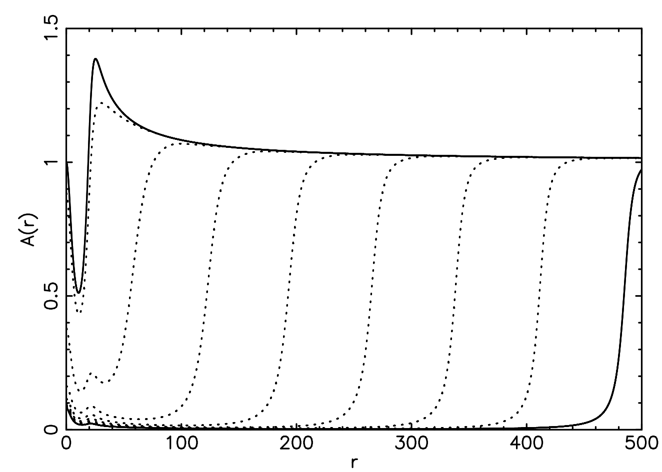

FIG. 4: Evolution of the radial metric function A.

IV. CONCLUSION

We conclude that the generic counterexample of the CCC proposed by HHM need not be such after all. It is of course still possible that a naked singularity might arise in their setting, but the question is fully open and as far as we see the only way to settle this issue is through a numerical simulation similar to the one we have carried out, but adapted to the SAdS asymptotics of the HHM example.

The CCC seems to resist attempts in both directions (prove or disprove) to change its nature of conjecture.

V. ACKNOWLEDGMENTS

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